TAKING ACCOUNT OF THE RELAXATION OF THE WETTING ANGLE IN THE CAPILLARY SUCTION OF LIQUID IN A GRAVITATIONAL FIELD

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An approximate solution is obtained for the differential equaiton describing the kinetics of capillary suction of liquid, taking account of inertial and gravitational forces, as well as the time dependence of the wetting angle.

An important role in the startup and switching-off of various thermal-engineering devices (thermal tubes, porous evaporative heat exchangers, etc.) is played by the initial stage of capillary suction. At present, the kinetics of liquid suction in the initial stage of its motion along a capillary has not been adequately studied. It is at this stage that calculation by the theoretical formulas gives the greatest discrepancy with experimental data [1-7]. This discrepancy may be explained in that inertial forces and the time dependence of the wetting angle are disregarded in solving the equations of capillary suction. Taking account of inertial forces in [2] led to considerable reduction in the discrepancy between theoretical calculation and experiment. By using the method outlined in [2], not only inertial, gravitational, and viscous forces may be taken into account, but also the time dependence of the wetting angle $\theta = f(t)$.

As shown in [5-7], at boundary angles $\theta \leq 90°$, the kinetics of capillary-wall wetting is satisfactorily described by an equation of relaxational type

$$\cos\theta = \cos\theta_r \left[1 - \beta \exp\left(-\frac{t}{\tau_r}\right)\right], \tag{1}$$

which is widely used in investigating capillary suction. Here β is a constant parameter depending on the initial wetting angle θ_0. Assuming that $t = 0$ in Eq. (1) gives

$$\beta = 1 - \frac{\cos\theta_0}{\cos\theta_r}. \tag{2}$$

When $\theta_0 = \theta_r$, $\beta = 0$ and $\cos\theta = \cos\theta_r = \text{const}$; when $\theta_0 = \pi/2$, $\beta = 1$ and $\cos\theta = \cos\theta_r[1 - \exp(-t/\tau_r)]$.

The physical basis of Eq. (1) was given in [8-12]. The relaxation time of the wetting process θ_r appearing in this equation depends on the properties of he liquid and the properties of the solid surface, on the wetting perimeter, etc. [5, 7, 9]. In a series of systems, τ_r is taken to be 0.5-1 sec [5, 7].

The dependence $\theta = f(t)$ must influence the kinetics of capillary suction, especially in the initial stages. In [5, 6, 9, 10], attempts were made to take account of the relaxation of the wetting angle in solving the equation of capillary suction, but inertial terms were neglected in the equations. In [1], the time dependence of θ was taken into account in an equation containing inertial terms, but for the case of a horizontal capillary or weightlessness.

In the general case, the complete equation of capillary suction taking account of Eq. (1) takes the form

$$\frac{d^2}{dt^2}(l^2) + a\frac{d}{dt}(l^2) + bl = c\left[1 - \beta\exp\left(-\frac{t}{\tau_r}\right)\right], \tag{3}$$

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where

$$a = \frac{8\eta}{r^2\rho}; \quad b = 2g\sin\varphi; \quad c = \frac{4\sigma\cos\theta_r}{r\rho}. \tag{4}$$

As in [2], the initial condtiions adopted for Eq. (3) are

$$l = 0; \quad \frac{dl}{dt} = \left(\frac{c}{2}\right)^{1/2} = \nu \ \text{ when } \ t = 0. \tag{5}$$

The solution of Eq. (3) is sought by the method described in [2].

The functions ℓ and ℓ^2 on the segment $[0, \ell_\infty]$ are represented in the form of Fourier series

$$l = \sum_{n=1}^{\infty} L_n, \quad l^2 = \sum_{n=1}^{\infty} \gamma_n L_n, \tag{6}$$

where L_n and γ_n are as follows

$$L_n = \alpha_n \sin\frac{n\pi l}{l_\infty}; \quad \gamma_n = l_\infty\left[1 - \frac{2}{(n\pi)^2}\left(1 - \frac{1}{\cos n\pi}\right)\right], \tag{7}$$

and α_n is the coefficient of the Fourier expansion; $\ell_\infty = 2\sigma\cos\theta_r/\rho gr$ is the equilibrium height of liquid ascent.

Substituting Eqs. (6) and (7) into Eqs. (3) and (5) leads to the following linear equation for L_n

$$\frac{d^2L_n}{dt^2} + a\frac{dL_n}{dt} + \frac{b}{\gamma_n}L_n = \frac{c}{n(n+1)\gamma_n}\left[1 - \beta\exp\left(-\frac{t}{\tau_r}\right)\right] \tag{8}$$

with the initial conditions

$$L_n = 0; \quad \frac{dL_n}{dt} = \frac{\nu}{n(n+1)} \ \text{ when } \ t = 0. \tag{9}$$

The general solution of Eq. (8) with the initial conditions in Eq. (9), as shown in [2], takes the form

$$L_n = \frac{l_\infty}{n(n+1)}\left\{1 - A_n\exp\left(-\frac{t}{\tau_r}\right) - \left[\frac{(1-A_n)\,m_{1n}\exp(m_{2n}t)}{m_{1n}-m_{2n}} - \frac{(1-A_n)\,m_{2n}\exp(m_{1n}t)}{m_{1n}-m_{2n}} + \frac{\left(\frac{}{l_\infty} - \frac{A_n}{\tau_r}\right)(\exp(m_{2n}t) - \exp(m_{1n}t))}{m_{1n}-m_{2n}}\right]\right\}, \tag{10}$$

where

$$A_n = \frac{\beta}{1 + \frac{\gamma_n}{6\tau_r^2}(1 - a\tau_p)};$$

$$m_{1n} = -\frac{a}{2}\left(1 - \sqrt{1 - \frac{4b}{a^2\gamma_n}}\right); \quad m_{2n} = -\frac{a}{2}\left(1 + \sqrt{1 - \frac{4b}{a^2\gamma_n}}\right). \tag{11}$$

Hence

$$l = \sum_{n=1}^{\infty} L_n = l_\infty\left\{1 - \exp\left(-\frac{t}{\tau_r}\right)\sum_{n=1}^{\infty}\frac{A_n}{n(n+1)} - \right.$$

$$-\sum_{n=1}^{\infty}\left[\frac{(1-A_n)(m_{1n}\exp(m_{2n}t)-m_{2n}\exp(m_{1n}t))}{n(n+1)(m_{1n}-m_{2n})}+\frac{\left(\frac{\nu}{l_\infty}-\frac{A_n}{\tau_r}\right)(\exp(m_{2n}t)-\exp(m_{1n}t))}{n(n+1)(m_{1n}-m_{2n})}\right]\Bigg\}. \quad (12)$$

Since the coefficients A_n, m_{1n}, and m_{2n} depend weakly on n, Eq. (12) may be written in the following approximate form

$$x=1-A\exp\left(-\frac{t}{\tau_r}\right)-\left\{\frac{(1-A)[m_1\exp(m_2t)-m_2\exp(m_1t)]}{m_1-m_2}+\left(\frac{\nu}{l_\infty}-\frac{A}{\tau_r}\right)\frac{\exp(m_2t)-\exp(m_1t)}{m_1-m_2}\right\}, \quad (13)$$

where

$$x=\frac{l}{l_\infty};\quad A=\beta\Big/\left(1+\frac{1-a\tau_r}{6\tau_r^2}\right);$$

$$m_1=-\frac{a}{2}\left(1-\sqrt{1-\frac{4b}{a^2l_\infty}}\right);$$

$$m_2=-\frac{a}{2}\left(1+\sqrt{1-\frac{4b}{a^2l_\infty}}\right).$$

When $\beta = 0$, $A = 0$, and the expression derived in [2] for the case when $\cos\theta$ = const is obtained from Eq. (13).

The results of calculations by Eq. (13) are compared wiht calculations by the formula in [2] and with experimental data for vertical glass capilaries (r = 0.24, 0.28, 0.32, 0.35 mm). The liquid investigated is distilled water. The experimental results are obtained by the method described in [2].

Calculation by Eq. (13) is possible if the parameter β is known. In the literature, it is sometimes tken to be unity, i.e., the initial angle $\theta_0 = \pi/2$. This condition evidently cannot be realized, since some concave meniscus is formed on account of wetting even at the initial instant of contact between the capillary wall and the liquid, so that $\theta_0 < \pi/2$ and $\beta < 1$. Since it is difficult to determine β experimentally at present, calculations by Eq. (13) are made for various β. According to these calculations, relaxation of the wetting angle

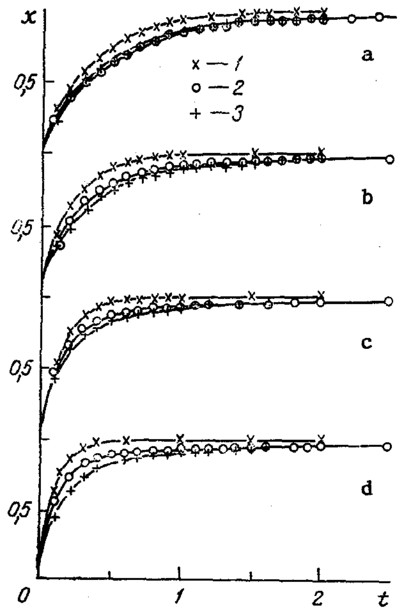

Fig. 1. Kinetics of capillary suction of distilled water in a vertical capillary: 1) calculation by the formula from [2]; 2) Eq. (13); 3) experimental data; r = $0.240\cdot10^{-3}$ (a), $0.285\cdot10^{-3}$ (b), $0.320\cdot10^{-3}$ (c), $0.350\cdot10^{-3}$ (d) m; x, dimensionless; t, sec.

has a pronounced influence on the kinetics of capillary suction; results which are closest to experiment are obtained when $\beta = 0.15$, which corrresponds to an initial boundary angle $\theta_0 = 34°$. Curves of the relative height of liquid ascent as a function of the suction time correspnding to this case are shown in Fig. 1 according to Eq. (13) and also experimental data for the given capillaries. For comparison, curves of $x = f(t)$ plotted from the formula in [2] for the case $\beta = 0$, i.e., $\theta = \theta_0 = \text{const}$, are also shown in Fig. 1.

It is evident from Fig. 1 that taking account of the time dependence of the wetting angle in the complete equation of capillary suction permits the most accurate description of the kinetics of liquid suction in a capillary in the early stages, as well as the determination of the initial wetting angle.

NOTATION

ℓ, displacement length of meniscus; ℓ_∞, maximum displacement length of meniscus; x, relative displacement length of meniscus; t, time of meniscus displacement; r, capillary radius; φ, angle of slope of capillary axis to horizontal; ρ, η, σ, density, kinematic viscosity, and surface tension of liquid; θ_0, initial wetting angle; θ_r, equilibrium wetting angle; τ_r, relaxation time of boundary wetting angle.

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